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Generic nonlinear processes in self-exciting dynamos and the long-term behaviour of the main geomagnetic field, including polarity superchrons

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When the main geomagnetic field undergoes a reversal in polarity it does so in no more than a few thousand years. Many hundreds of such reversals must have occurred over geological time (the age of the Earth being *ca.* 4500 Ma), with the highly irregular intervals between reversals ranging in duration from *ca.* 0.25 Ma to *ca.* 30 Ma. During the past *ca.* 400 Ma, the period studied most intensively by palaeomagnetic workers, there have been two so-called 'superchron' intervals, namely the Permian superchron from *ca.* 290 to *ca.* 260 Ma ago, when a magnetic compass would have pointed south, and the Cretaceous superchron from *ca.* 110 to *ca.* 80 Ma ago, when the polarity was the same as it is now, and there may have been many such intervals at earlier times. The geomagnetic field is generated in the Earth's liquid metallic outer core, where the electrical conductivity is high enough (but not too high) for efficient self-exciting magnetohydrodynamic (MHD) dynamo action to take place. One generic process in all self-exciting dynamos is the redistribution of kinetic energy within the system by the action of Lorentz forces. Such forces operating within a self-exciting Faraday disc homopolar dynamo loaded with a nonlinear series motor can quench fluctuations, thereby promoting steady dynamo action over a wide range of conditions, with no reversals in the direction of the dynamo current. If this recently discovered process of nonlinear quenching occurs in the MHD geodynamo, it will provide a firm basis for understanding superchrons and other features of the long-term behaviour of the main geomagnetic field. This hypothesis implies that in the inhibition of polarity reversals a crucial role is played by those eddies in the liquid outer core that are driven mainly by Lorentz forces, and it could be tested by analysing the kinetic energy spectrum of eddies in valid numerical geodynamo models. Also crucial, and testable, is the role played in the stimulation of reversals by very slow changes in the lateral boundary conditions at the overlying core–mantle boundary associated with intermittent convection and other dynamical processes occurring in the Earth's lower mantle.

Keywords: magnetohydrodynamics; nonlinear quenching; self-exciting dynamos; geomagnetism; polarity superchrons

1. Introduction

I was pleased to be asked to take part in this timely Royal Society Discussion Meeting on the long-term behaviour of the main geomagnetic field, as revealed by extensive palaeomagnetic studies of the history of the field recorded in igneous and sedimentary rocks (see, for example, Jacobs 1994; Fuller *et al*. 1996; Opdyke & Channell 1996; Gallet & Hulot 1997; Thomas *et al*. 1998; Love 1998; Gubbins 1998, 1999; Merrill & McFadden 1999; see also the other papers in this issue). Progress towards the interpretation of this behaviour is inseparable from advances in our understanding of the structure and dynamics of the Earth's deep interior and of magnetohydrodynamic (MHD) processes in rapidly rotating planets and stars. These are areas of geophysics and fluid dynamics within which I have worked from time to time over a number of years, and upon which findings outlined in this paper of recent studies of self-exciting Faraday disc homopolar dynamos (Hide 1995; Hide *et al*. 1996; Hide 1997, hereafter H97; Hide 1998, hereafter H98; Hide & Moroz 1999, hereafter HM99) may have some bearing.

2. Core motions and the geodynamo; some working hypotheses

As attested by many papers in the geophysical literature (such as those in this issue by Bloxham, Coe *et al*., Glatzmaier *et al*., Jones, Kono, and Sarson), theoretical research on the origin of the main geomagnetic field continues to be guided by the following working hypotheses:

(a) that the main geomagnetic field is a manifestation of electric currents flowing within the Earth's deep interior, and that these currents are generated by self-exciting dynamo action (a process first proposed by Larmor in connection with solar magnetism) involving motional induction associated with buoyancy-driven MHD flow in the Earth's liquid, metallic (iron) outer core (Frenkel 1945; Elsasser 1947);

(b) that within the core the geomagnetic field has a 'toroidal' part which, unlike the 'poloidal' part, has lines of force that are confined to the core, and which may be so much stronger on average than the 'poloidal' field there that it accounts for most of the total energy of the field (Elsasser 1947);

(c) that the near alignment of the geomagnetic dipole with the Earth's rotational axis is associated with the influence on core motions of Coriolis forces due to the Earth's rotation (Elsasser 1939), which render the patterns of core flow highly anisotropic (cf. Hide 1988);

(d) that Coriolis forces render core flow sensitive to the presence of the underlying solid inner core (Hide 1953, 1966), which gives rise to a detached shear layer across which mixing is strongly inhibited in the vicinity of the imaginary cylindrical surface that is tangential to the equator of the inner core and intersects the core–mantle boundary (CMB) at latitudes near $\pm 67°$;

(e) that Coriolis forces also render core flow sensitive to slight deformations in the shape of the CMB as well as to lateral variations in thermal and related (e.g. electromagnetic) conditions prevailing at the CMB (Hide 1967), all produced by slowly varying dynamical processes operating on geological time-scales within the overlying mantle.

The detached shear layers referred to in (d) above were produced during experiments on (buoyancy-driven) sloping convection in a spherical annulus. Similar layers in mechanically driven homogeneous fluids were later studied by several workers (see Greenspan 1968). Subsequently termed 'Stewartson layers' (by me), they have their counterparts in flows found in rotating MHD systems, including numerical geodynamo models (see, for example, Zhang & Busse 1989; Soward 1992; Hollerbach 1996; Glatzmaier & Roberts 1995, 1997; Olson *et al*. 1999; and this issue).

Such detached shear layers occurring within the Earth's liquid core would stretch lines of force of the poloidal part of the geomagnetic field, thereby providing the main source of the toroidal field (see item (b) above), which at levels well below the CMB may equilibrate at a mean strength for which Lorentz forces are typically in 'magnetostrophic' balance with Coriolis forces. Magnetostrophic flows, which are of central importance in geophysical and astrophysical fluid dynamics, include an important class of highly dispersive waves with periods *directly* proportional to the rate of rotation of the system and inversely proportional to the *square* of the Alfvén speed.

3. Self-exciting dynamos

Self-exciting dynamos are electromechanical engineering devices or naturally occurring MHD fluid systems that convert mechanical energy into magnetic energy without the aid of permanent magnets. They differ widely in their details, but they all share the following essential characteristics:

(a) the mechanical-to-magnetic energy conversion process is due to motional induction (as represented by the nonlinear term $\mathbf{u} \times \mathbf{B}$ in the equations of MHD, where $\mathbf{u}$ is the Eulerian flow velocity at a general point and $\mathbf{B}$ the magnetic field), and it starts with the amplification of any infinitesimally weak adventitious magnetic field;

(b) for the amplification process to work, motional induction must overcome ohmic losses, implying that the electrical resistance of the system must be sufficiently low (i.e. a sufficiently high magnetic Reynolds number

$$R = UL\mu\sigma \tag{3.1}$$

in MHD dynamos, where U is a characteristic flow speed, L a characteristic length, and μ and σ are respective measures of the magnetic permeability and electrical conductivity of the fluid);

(c) for the magnetic field to be able to diffuse into the surrounding medium, the electrical resistance must not be *too* low (and this sets an *upper* limit on R in MHD dynamos);

(d) Lorentz forces (as represented by the nonlinear term $\mathbf{j} \times \mathbf{B}$ in MHD dynamos, where $\mathbf{j}$ is the electric current density) redistribute kinetic energy within the system (thereby retarding the buoyancy-driven eddies in typical MHD dynamos such as the geodynamo and accelerating motions in other parts of the eddy spectrum);

(e) no matter how weak, mechanical friction (viscosity in MHD dynamos), which *inter alia* dissipates kinetic energy, is never negligible;

(f) internal coupling and feedback (as represented by the terms $\mathbf{u}\times\mathbf{B}$ and $\mathbf{j}\times\mathbf{B}$ in MHD dynamos) give rise to behaviour characteristic of nonlinear systems (i.e. sensitivity to initial conditions leading to non-uniqueness (sometimes called 'multiple solutions' or 'multiple equilibria'), large amplitude fluctuations (including 'deterministic chaos'), hysteresis, nonlinear stability, etc.).

The combination of characteristics (b) and (c) is important when defining exactly what is meant by self-exciting dynamo action. It effectively excludes systems that amplify magnetic energy by motional induction but are incapable of increasing the linkage N (say) of magnetic lines of force with the outer boundary of the system. The time rate of change of N is inversely proportional to $\mu\sigma$ and has its sign and magnitude determined entirely by the detailed structure of $\mathbf{B}$ in the vicinity of the boundary, including the length-scale associated with normal gradients of $\mathbf{B}$ (Hide 1981). A substantial body of work now exists on so-called 'fast dynamos' (see Childress & Gilbert 1995) based on mathematical simplifications afforded by the singular limit of nearly infinite R (see equation (3.1)). Such dynamos certainly satisfy requirement (b), but they do not appear to satisfy (c).

Characteristics (b) and (c) also bear on the interpretation of the magnetic fields of other planets. The observed fields of Jupiter and Saturn may be generated by self-exciting dynamo action taking place within the lower reaches of their mainly molecular hydrogen outer layers, rather than within their more highly conducting metallic hydrogen cores where R should be high enough to satisfy (b) but could be too high to satisfy (c) (Hide 1965). This proposal had implications for the magnetic fields of the other outer planets and it subsequently gained support not only from determinations of effects of pressure, temperature and chemical 'impurities' on the electrical conductivity of hydrogen but also from the discovery of the magnetic fields of Uranus and Neptune, objects not quite large enough for their main constituents to exist in compressed metallic states.

MHD dynamos are governed by four-dimensional (space and time) nonlinear *partial* differential equations (PDEs) expressing the laws of mechanics, thermodynamics and electrodynamics, to be solved under realistic boundary conditions. These equations are barely tractable at present, even with the aid of the most powerful computers available, although significant progress in this direction has been made in recent years and much more can be expected in the future (for references see, for example, Glatzmaier & Roberts 1995, 1997; Zhang & Jones 1997; Zhang *et al*. 1998; Olson *et al*. 1999; Sakuraba & Kono 1999; and various papers in this issue). So the full PDEs have yet to be used in investigations of the time-series of polarity reversals. More promising for elucidating temporal behaviour are 'intermediate' models of MHD dynamos in which the nonlinear PDEs are simplified by carrying out spatial averages, as in a variety of mean field dynamos or multiple scale dynamos (see, for example, Moffatt 1978; Parker 1979; Ghil & Childress 1987; Zhang & Busse 1989; Braginsky 1991; Soward 1992; Krause 1993; Proctor & Gilbert 1994; Hirsching & Busse 1995; Hollerbach & Jones 1995; Jones *et al*. 1995; LeMouël *et al*. 1997; Sarson & Jones 1999).

4. Self-exciting Faraday disc homopolar dynamo loaded with a nonlinear motor

An even simpler way to investigate temporal behaviour of self-exciting dynamos is the one exploited in the present paper. It is based on detailed analyses of the nonlinear *ordinary* differential equations (ODEs, see equations (4.2) below) in the single independent variable time, t (say), which govern the behaviour of engineering devices such as the Faraday disc homopolar generator. (For references to extensive literature based on pioneering studies by Bullard, Rikitake, Malkus and others starting in the 1950s, see Ghil & Childress (1987), Ershov *et al.* (1989), Jacobs (1994), Turcotte (1992), Dubois (1995), Hide (1995), Hide *et al.* (1996), H97, H98, Plunian *et al.* (1998), HM99.) Apart from the undoubted mathematical interest of the solutions of the governing ODEs, the findings of those investigations that treat physically realistic systems—and we must emphasize here that this requirement excludes all friction-free systems (Hide 1995; cf. characteristic (e) of § 3 above)—provide general insights into the likely behaviour of the more complex MHD systems.

An obvious strategy for discovering generic processes in self-exciting dynamos is to start by investigating the behaviour of simple (but not oversimplified) systems and then, in the light of the results thus obtained, formulating and executing suitable diagnostic tests of more complex MHD systems. The simple system exploited here (see H97; H98; HM99)—to my knowledge, the first such system to satisfy *all* the criteria listed above in § 3—comprises the usual Faraday disc and coil arrangement loaded with a nonlinear electric motor connected in series with the coil. The motor is a crucial additional element in the circuit, for it allows Lorentz forces to redistribute kinetic energy. Such a system could be made in the laboratory, but it would be large and expensive. It is cheaper and easier to solve the governing mathematical equations, where necessary making use of standard digital computers or dedicated electronic analogue circuits (see H98).

The disc is driven into rotation with dimensionless angular speed $y(t)$ by a steady applied couple proportional to the dimensionless parameter a, which is inversely proportional to the moment of inertia of the disc (see equations (4.2) below). (Here t is measured in units of the ratio of the self-inductance of the coil to the total electrical resistance of the dynamo circuit.) Retarding the motion of the disc is a frictional couple $-ky(t)$ as well as a Lorentz couple $-ax(t)v(t)$, where $x(t)$ is the main electric current generated by the dynamo and $v(t)$ is the magnetic flux linkage of the disc. (In these dimensionless units $v(t)$ is equal to $x(t)$ plus a term proportional the azimuthal eddy current that is induced in the disc when $\mathrm{d}x/\mathrm{d}t \neq 0$.) In the absence of Lorentz forces, when friction alone retards the motion of the disc, y has the steady value a/k (see equation (5.1)).

The armature of the motor is driven into rotation with angular speed $z(t)$ relative to the stationary ambient magnetic field within the motor by a Lorentz couple $x(t)f(x(t))$ produced by the dynamo current, and it is retarded by a frictional couple $-lz(t)$ (see equation (4.2 *d*)). Here

$$f(x(t)) = 1 - e + esx(t), \qquad 0 \leqslant e \leqslant 1, \tag{4.1}$$

which depends on the design of the motor, specifies the stationary ambient magnetic field. The first and second terms on the right-hand side of equation (4.1) are in the ratio $(1-e)$:$esx(t)$, so the parameter e in that equation is a measure of the nonlinearity of the electromechanical characteristics of the motor, which is linear only in the special case when $e = 0$. The contribution $esx(t)$ is produced by diverting the dynamo current through stationary field windings (s being a measure of the mutual inductance between the armature and the field windings). This is complemented by the contribution equal to $(1-e)$ provided by an ‘outside source’, which could be a permanent magnet. But it is important to stress that this is not the only possibility, for the outside source could be the current in the coil of another ('secondary') self-exciting dynamo placed near the motor of the 'primary' dynamo. This perfectly feasible arrangement provides a *Gedanken* demonstration (which could if necessary be elaborated mathematically) that cases of geophysical or astrophysical interest, where permanent magnetism is usually of no significance, are *not* restricted to values of e equal to unity (see §§ 6 and 7 below).

The four-mode dimensionless autonomous set of nonlinear ODEs that governs this system is the following:

$$\dot{x} + m\dot{v} = (1 + m)[-x + yv - bzf(x)], \tag{4.2 a}$$

$$n\dot{v} = x - v, \tag{4.2 b}$$

$$\dot{y} = a(1 - xv) - ky, \tag{4.2 c}$$

$$\dot{z} = xf(x) - lz, \tag{4.2 d}$$

where $\dot{x} = \mathrm{d}x/\mathrm{d}t$, etc., and $f(x)$ is given by equation (4.1). The essentially non-negative control parameters (a, k, n, m) specify the electromechanical characteristics of the disc, while (b, l, s, e) specify those of the motor. The parameter n is inversely proportional to the electrical resistance of the disc to the flow of azimuthal eddy currents; m is proportional to the square of the mutual inductance between the disc and coil and inversely proportional to the difference between the product of the self-inductances of the disc and coil and the square of their mutual inductance; b is proportional to the self-inductance of the armature of the motor and inversely proportional to its moment of inertia.

Equations (4.2 a), (4.2 b), respectively, express Kirchhoff's laws applied to the dynamo current $x(t)$ flowing in the the main circuit and to the induced azimuthal eddy current flowing in the disc. Equations (4.2 c), (4.2 d), respectively, express angular momentum considerations applied to the motion of the disc and to the motion of the armature of the motor. When $e = 0$ or $e = 1$, for every solution (x, y, z, v) of these equations there are solutions with exactly reversed currents (magnetic fields), respectively $(-x, y, -z, -v)$, and $(-x, y, z, -v)$, but *exact* reversal is not a property of any of the solutions when $0 < e < 1$ (see equation (4.1) above and § 7 below).

5. Classification of solutions and nonlinear quenching

Equations (4.2) can be investigated by standard methods involving general considerations of their structure, stability and bifurcation analysis, and direct numerical integration. Owing to nonlinearity, solutions in which we are mainly interested, namely those that persist after initial transients have died away, can be very sensitive to the conditions imposed on (x, y, z, v) at time $t = 0$. The full exploitation of the equations will be a lengthy undertaking, for in each case studied it is necessary to specify every control parameter $(a, k, n, m; b, l, s, e)$. These can be represented by a point in eight-dimensional 'parameter space', but certain key findings to date are readily outlined with the aid of a two-dimensional regime diagram with $\bar{b} = b/l$ as abscissa and $\bar{a} = a/k$ as the ordinate (see figure 1).

There is one 'trivial' equilibrium solution, namely

$$(x, y, z, v) = (0, \bar{a}, 0, 0), \tag{5.1}$$

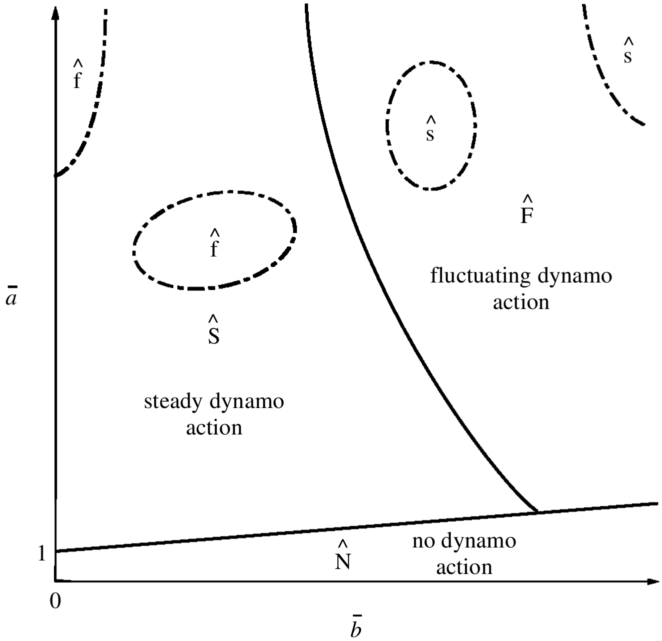

Figure 1. Schematic regime diagram with the dimensionless parameters $\bar{b} = b/l$ as abscissa and $\bar{a} = a/k$ as ordinate (see equations (4.2)). The parameter $\bar{a}$ depends on the electromechanical properties of the disc and on the magnitude of the steady applied couple that drives the disc into rotation, to which $\bar{a}$ is directly proportional; $\bar{b}$ depends on the properties of the motor. Self-exciting dynamo action takes place when $\bar{a} > \bar{a}^*$ (where $\bar{a}^* = \bar{a}^*(\bar{b}; k, n, m; l, s, e)$), a criterion satisfied within region Ŝ and subregions ŝ where steady dynamo action occurs, and also within region F̂ and subregions f̂ where fluctuating dynamo action occurs. The extent to which fluctuations are inhibited by nonlinear quenching depends mainly on the dimensionless parameter e (see equation (4.1)). It is within region N̂, where $0 < \bar{a} < \bar{a}^*$, that $\bar{a}$ is too small for dynamo action to be possible.

where $\bar{a}$ corresponds to the magnetic Reynolds number R in MHD dynamos (see equation (3.1)). This is the only equilibrium solution within those regions N̂ (say) of parameter space where $\bar{a}$ does not exceed the critical value $\bar{a}^*(k, n, m; b, l, s, e)$ (say), and because this solution is stable when $\bar{a} < \bar{a}^*$, persistent dynamo action cannot occur within N̂. It is throughout the rest of parameter space, regions Ŷ (say) where $\bar{a} > \bar{a}^*$ and the trivial solution is unstable, where persistent dynamo action does take place.

Within these regions Ŷ there are two general possibilities, namely *steady* dynamo action and *fluctuating* dynamo action. The first occurs within regions Ŝ and subregions ŝ (say) (see figure 1), where (x, y, z, v) are given by one or other (depending on initial conditions) of two 'non-trivial' equilibrium solutions of equations (4.2) (i.e. those with $(\dot{x}, \dot{y}, \dot{z}, \dot{v}) = (0, 0, 0, 0)$ but $(x, y, z, v) \neq (0, \bar{a}, 0, 0)$), which are stable within Ŝ and ŝ. Fluctuating dynamo action occurs throughout the rest of Ŷ, within regions F̂ and subregions f̂ (say), where the 'non-trivial' equilibrium solutions lose their stability. Persistent solutions of equations (4.2) then exhibit large amplitude fluctuations of varying degrees of complexity characteristic of nonlinear oscillations, including multiple periodicity and chaos.

In the simple but physically realistic case when $n = 0$, the parameter m is redundant and $v = x$. When in addition e is closer to zero than it is to unity (see equation (3.1)) the system exhibits rich temporal behaviour over extensive regions $\hat{\mathrm{F}}$ and $\hat{\mathrm{f}}$ (see figure 1). According to a detailed analytical and numerical study of the case when $e = 0$ (Hide *et al.* 1996), $\bar{a}^*$, which defines the upper boundary of region $\hat{\mathrm{N}}$, satisfies

$$\bar{a}^* = \min(1 + \bar{b}, 1 + l). \tag{5.2}$$

Steady dynamo action with

$$x = \pm[1 + (1 + \bar{b})/\bar{a}]^{1/2} = v, \qquad y = 1 + \bar{b}, \qquad z = x/l, \tag{5.3}$$

is found in that part of the diagram lying above the straight line $\bar{a} = 1 + \bar{b}$ and to the left of the curved line

$$\bar{a} = 1 + l(2\bar{b} - k - l)/2(k - \bar{b}) + 3\bar{b}/2 \tag{5.4}$$

extending from the (Takens–Bogdanov double-zero eigenvalue) point where $(\bar{b}, \bar{a}) = (l, 1 + l)$ at its lowest end and tending asymptotically to the line where $\bar{b} = k$ when $\bar{a} = \infty$. Fluctuating solutions of large amplitude and exhibiting varying degrees of complexity, including multiple periodicity and chaos, occur throughout the extensive region lying to the right of the curved line of Hopf bifurcations given by equation (5.4) and above the line $\bar{a} = 1 + l$.

The remarkable and unexpected discovery revealed by the analysis reported in H98 is that when $e = 1$ (rather than zero, see equation (4.1)) all fluctuations are quenched, leaving just two regimes in the $(\bar{b}, \bar{a})$ diagram, $\hat{\mathrm{N}}$ and $\hat{\mathrm{S}}$, with no dynamo action occurring within the former and steady dynamo action within the latter, with (x, y, z, v) satisfying

$$x = \pm[(\bar{a} - 1)/(\bar{a} + s^2\bar{b})]^{1/2} = v, \qquad y = \bar{a}(1 + s^2\bar{b})/(\bar{a} + s^2\bar{b}), \qquad z = sx^2/l. \tag{5.5}$$

The boundary between $\hat{\mathrm{N}}$ and $\hat{\mathrm{S}}$ then occurs where $\bar{a} = \bar{a}^* = 1$.

Subsequent work (HM99) shows that this process of nonlinear quenching of dynamo fluctuations is also effective even when n is non-zero provided that e is close to unity. Regimes of fluctuating dynamo action then appear, but they occupy very limited 'subregions' $\hat{\mathrm{f}}$ of $(\bar{b}, \bar{a})$ parameter space, mainly where $\bar{b}$ is very small and $\bar{a}$ very large. Nonlinear quenching also occurs at intermediate values of e; regions $\hat{\mathrm{F}}$ and subregions $\hat{\mathrm{f}}$, where fluctuating dynamo action occurs are then more extensive than when $e = 1$, but less extensive than when $e = 0$.

Insight into the essential role of the motor in both the production and quenching of fluctuations can be obtained by considering the motor as an oscillator, with the necessary inertia provided by the moving armature and the restoring couple by Lorentz forces acting on the armature. When dissipation due to ohmic and frictional effects is negligible the response of an isolated motor to an initial disturbance (x_0, z_0) satisfies the following equations (derivable by inspection from equations (4.2)):

$$\dot{x} = -bzf(x) \quad \text{and} \quad \dot{z} = xf(x). \tag{5.6}$$

These have periodic solutions with trajectories in the (x, z) phase plane lying on ellipses $x^2 + bz^2 = x_0^2 + bz_0^2$. The solutions exhibiting the most pronounced persistent oscillations are the harmonic ones of angular frequency $b^{1/2}$ found when $e = 0$ (so that $f(x) = 1$). At intermediate values of e the system can still oscillate periodically but more slowly and non-harmonically and with smaller amplitude than in the case when $e = 0$. Oscillations are completely quenched when $e = 1$ (so that $f(x) = sx$); persistent solutions are then steady with $(x, z) = (0, (z_0^2 + x_0^2/b)^{1/2})$.

The production of large-amplitude chaotic fluctuations in nonlinear systems has been the subject of many mathematical investigations. Further studies of autonomous sets of ODEs exemplified by equations (4.2) could shed additional light on mechanisms capable of suppressing chaos.

6. Time-series of geomagnetic polarity reversals

In the light of the phenomenon of nonlinear quenching, we turn now to the problem of interpreting the geomagnetic polarity record in terms of feasible behaviour of the MHD geodynamo operating in the Earth's liquid metallic outer core.

Very slow and intermittent convection and other dynamical processes taking place within the lower reaches of the 'solid' mantle overlying the liquid core must give rise to slow fluctuations, on geological time-scales, in the lateral variations of the conditions prevailing at the CMB. These cause changes in both spatial and temporal characteristics of core motions and, consequently, in the magnetic fields they produce (see Hide 1967; Jones 1977; Courtillot & Besse 1987; Loper 1997; Lay *et al.* 1998; Gubbins 1998; also § 2 above). So it is not unreasonable to suppose that each polarity superchron could be associated with a long quiescent period when convection in the lower mantle (but not necessarily in the upper mantle) is comparatively feeble.

Because the CMB would then be relatively undisturbed, the corresponding state of the geodynamo would be highly stable owing to processes analogous to the nonlinear quenching mechanism found in disc dynamos. What corresponds within the core to the presence of the motor in the disc dynamo with a Lorentz torque proportional to the square of the electric current (when $e = 1$) is that part of the spectrum of core motions contains eddies that are driven mainly by Lorentz forces (H97, H98)—rather than being driven directly by the largely global scale buoyancy forces that generate flow in the core, or by mechanical interactions with eddies on other scales of motion. Lorentz forces also 'load' the geodynamo by exerting couples on the highly conducting solid inner core and on the weakly conducting lower mantle (H97).

During intensive phases of intermittent mantle convection, conditions at the CMB become disturbed, with viscous stresses in the lower mantle distorting the shape of the CMB and with buoyancy forces associated with increased lateral temperature gradients in the lower mantle now contributing to the driving of core motions. Concomitant distortions in the flow patterns and magnetic fields within the core (see § 2) would stimulate changes in the main geodynamo, possibly by placing it within regimes corresponding to those found in the single disc dynamo when the parameter e is no longer close to unity. As we have seen, nonlinear quenching is then much less effective than when $e = 1$, and frequent reversals as well as less extreme fluctuations (e.g. geomagnetic excursions) are more likely to occur.

Possibly the simplest conceivable concrete example of a system capable of such behaviour is the one outlined in that part of § 4 above, namely a 'primary' single disc dynamo subject to *steady* forcing producing the primary magnetic field and interacting with a 'secondary' single disc dynamo subject to *slowly varying and intermittent* forcing. The secondary dynamo interacts with the primary dynamo by modulating the background magnetic field near the motor of the primary dynamo. When this background field is weak the value of e for the primary dynamo is close to unity, so that reversals and other fluctuations are inhibited by nonlinear quenching. But as the background field increases in strength the effective value of e for the primary dynamo drops to values well below unity, thereby stimulating frequent reversals and other time dependence in the primary field.

Typical time-series of $x(t)$ for the primary dynamo would include 'superchrons' of fixed 'polarity'—positive or negative depending on initial conditions—during those intervals when the secondary dynamo is relatively inactive and e is consequently close to unity. Highly time-dependent behaviour found between superchrons occurs when the secondary dynamo is active and the corresponding value of e for the primary dynamo is significantly less than unity. For further details see for example the time-series displayed in figs 1 and 2 of H98, figs 8 and 9 of Hide *et al.* (1996), or figs 5.1–5.5 of HM99.

7. Excursions, biases and symmetry considerations

Noteworthy features of typical time-series of $x(t)$ include fluctuations which are more regular than reversals, less pronounced in amplitude, and exhibit a systematic monotonic build up in amplitude before each reversal. They are reminiscent of the so-called 'excursions' seen in the geomagnetic record which, according to the ideas presented here, should be less pronounced during polarity superchrons than at other times, a point which could be checked against observations by palaeomagnetic workers.

Other points indicated by our model are (a) that the maximum strength of the geomagnetic field (and possibly the RMS average strength) could be systematically higher when reversals are frequent than when they are infrequent, and (b) that during an interval of frequent reversals the average strength of the field when the polarity is 'normal' could differ systematically from the average strength when the polarity is 'reversed'.

All self-exciting dynamos, be they disc dynamos or MHD dynamos, satisfy essentially nonlinear equations, with generic solutions that are multiple and much more varied and interesting than just the matching pairs $(\mathbf{u}, \mathbf{B})$ and $(\mathbf{u}, -\mathbf{B})$ corresponding to an unaltered velocity field and a completely reversed magnetic field. In discussing the implications of symmetry properties of the governing MHD equations for possible biases to be expected in geomagnetic time-series it is necessary to define the length of time, T (say), taken for any bias in the time-series of $\mathbf{B}$ to vanish. T is clearly infinite in the case of stable steady persistent solutions, where the sign of $\mathbf{B}$ after transients have died away is determined entirely by initial conditions and never changes thereafter. On the other hand, for fluctuating persistent solutions associated with instability, T can of course be finite, but significantly—according to general mathematical studies of nonlinear time-series (see, for example, Smith 1993)—this need not be true in all cases. If T is less than the age of the Earth, then the observed time-series of polarity reversals would show no net bias, but not otherwise. This is a matter for further investigation rather than *a priori* assessment.

What palaeomagnetic observations indicate is that the polarity of *part* of $\mathbf{B}$ near the Earth's surface changes sign; but they do *not* of course show that the whole of $\mathbf{B}$ reverses exactly everywhere, and there are no observations of the velocity field

u whatsoever. The theoretical possibility of matching solutions $(\mathbf{u}, \mathbf{B})$, $(\mathbf{u}, -\mathbf{B})$, though important, should not be overemphasized in attempts to interpret the long-term behaviour of the geomagnetic field.

8. Concluding remarks

Geodynamo studies facilitate the exploitation of geomagnetic and other geophysical data in research on the structure, dynamics and evolution of the Earth's deep interior, about which we still have much to learn. As in other areas of geophysical fluid dynamics (e.g. dynamical meteorology and oceanography), such work is impeded by the intractability of the governing nonlinear partial differential equations—compounded in this case by a lack of detailed geomagnetic observations covering long periods of time and by the technical difficulties of carrying out laboratory experiments in magnetohydrodynamics. So the findings of thorough investigations of much simpler but physically realistic systems can still be important in geodynamo research, and it is in this spirit that the new proposals outlined in this paper are made.

The interpretation of polarity superchrons, chrons and subchrons, and of excursions and other features of the irregular time-series of long-term fluctuations in the main geomagnetic field, must rank as a major problem in modern geophysics. The process of nonlinear quenching may hold the key to its solution.

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